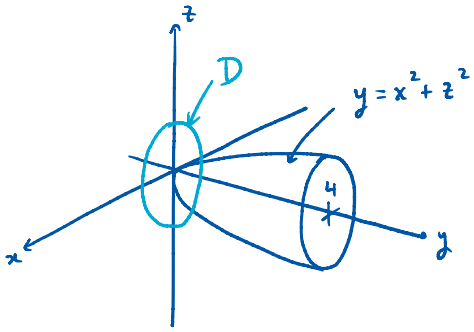
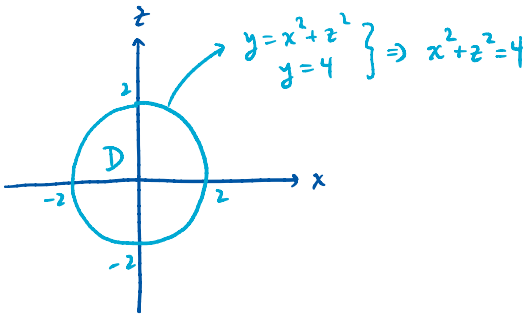
3 If we have time...

Example 7. Express $\iiint_E \sqrt{x^2+z^2}\,dV$ as an iterated integral, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.





Naturally expressed as a Type C 3D region:

$$\iiint_E \sqrt{x^2+z^2}\,dV$$

$$= \iint_D \left[\int_{x^2+z^2}^{4} \sqrt{x^2+z^2}\,dy \right] dA$$

$$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+z^2}^{4} \sqrt{x^2+z^2}\,dy\,dz\,dx$$

(D as Type I)

$$= \int_{-2}^{2} \int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}} \int_{x^2+z^2}^{4} \sqrt{x^2+z^2}\,dy\,dx\,dz$$

(D as Type II)